Imaging density in the Earth and the construction of optimal observables

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1. Synthetic inversion setup & strategy

We perform synthetic waveform tomography experiments in 2-D using the adjoint method in a mantle-scale model (Fig. 2).
- Whole mantle setup
- 8 point force sources at (x) 56 km depth
- 16 receivers (o) at the surface
- Absorbing boundaries left and right
- Bottom boundary reflecting (a core-mantle boundary)

The target model is known (Fig. 3). In this model, density, S-velocity and P-velocity are uncorrelated by design. This is because we want to image density independently without any prior constraints about its geometry and distribution. We investigate the following questions:
- Can density be imaged as a separate, independent parameter?
- What is the effect of ignoring density when density structure is present?
- What is the effect of (extraordinarily) scaling density to S velocity?
- Can density be imaged in the absence of noise?

2. Synthetic results

(a) Target model
(b) Recovered model after 16 iterations when all three parameters: density, S velocity and P velocity are free. Density is still recovered at the edges of the anomaly. Sensitivity to density has a significant effect (Fig 2a).
(c) Recovered model for the inversion where only S velocity and P velocities are unconstrained. Density remains fixed at PREM values. The resulting density structure maps into the other parameters as circular anomalies, mainly at the edges of the locations of the actual density anomalies.
(d) Recovered model for the inversion where only density and P velocity are free. Density is still recovered at the edges of the anomalies. Sensitivity to density is optimised when observable 1...this is done, finding the optimal b (a non-linear, but rather cheap problem) can be done with a simple grid search.

3. Towards Optimal Observables

Despite the fact that density can clearly be recovered (box 2), the density effect on waveforms remains weak (Fig 1h), and trade-offs persist (e.g. Fig 4d). The method of Optimal Observables as developed by Bernauer et al (GJI 2014) is excellently suited to address this issue of trade-offs.

(a) Target model in density, where each column is scaled to S velocity (small panel) according to a different scaling- each of these within a reasonable range of Earth-like values.
(b) The recovered density model where all parameters are free (like in the reference case above). Here, density is best recovered where it is simple, and because of the overlapping structures, strong artefacts are present all throughout the model domain.
(c) Recovered density model if it is scaled to S velocity with fixed R_p = 0.2. The middle column is here correct, and there are much fewer artefacts. However, all the interesting information on the two other columns, whose scaling deviates from the imposed value, is completely lost. In this case, more artefacts are present in the recovered P model.

4. Theory of Optimal Observables

We define a joint optimisation problem in which we try to maximise sensitivity to one parameter (expressed as sensitivity power) whilst minimising sensitivity to the other two. This is expressed in the weighting vector b and the optimality criterion.

\[ R_{\rho/S} = b_1 \delta \rho + b_2 \delta v_S + b_3 \delta v_P \]

(b) The recovered density model when imposing a fixed scaling R_p = 0.2. The middle column is here correct, and there are much fewer artefacts. However, all the interesting information on the two other columns, whose scaling deviates from the imposed value, is completely lost. In this case, more artefacts are present in the recovered P model.

5. Conclusions

Questions?

6. Eastern Mediterranean

As a study area, we chose the Eastern Mediterranean, a tectonically active region with good data coverage. For one source-receiver path, we show two windows on the Z trace, filtered between 50-150 s. We also include the time-frequency phase misfit (Fichtner et al, GJI 2008) sensitivity kernels. Here, a number of things become apparent:
- Amplitudes are lower in window 1 (body waves), but sensitivity to density is larger than in window 2 (surface waves).
- Sensitivity to density has a significantly different pattern from sensitivity to the other parameters. For this reason, we will construct optimal observables for density using different windows per trace, and different frequency bands.

7. Toy problems with two observables

Scenario 1: For any b, sensitivity to density is optimised when observable 1 gets full weight and 2 gets none. This is because p is linearly independent from the other parameters.

Scenario 2: Now the observable weights depend on the choice of b. As in realistic applications, parameters are not linearly independent - resulting in variable optimal observables.

The most expensive step is calculating the kernel-kernel products required for the sensitivity power. Once this is done, finding the optimal b (a non-linear, but rather cheap problem) can be done with a simple grid search.

References:

Sensitivity kernels for P velocity and density.