Transport in an idealized three-gyre system with application to the Adriatic Sea

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What controls intergyre transport in a perturbed three-gyre system?

Introduction

Motivated by observations of surface drifters in the Adriatic Sea (see above), transport in a threegyre system is studied with the aid of dynamical systems techniques. The velocity field is assumed to be two-dimensional and incompressible, and composed of a steady three-gyre background flow on which a time-dependent perturbation is superimposed. Two systems of this type are considered: 1) an analytical model of the Adriatic Sea; and 2) a numerically-based altimetry derived model of the Adriatic Sea. It is shown that a new phenomenon arises in a three-gyre system, which is not present in a two-gyre system. Due to this phenomenon, the three-gyre system has qualitatively different transport properties for small and large perturbations to the background. For a small perturbation to the gyres exchange no fluid with the third gyre. When the perturbation strength exceeds a certain threshold, transport between all three gyres occurs.

Dynamical systems theory

1 Lagrangian equations of motion
2 Action-angle variables (I, \theta)
3 Kolmogorov-Arnold-Moser (KAM) theorem
4 Strong KAM stability near shearless torus
5 Simulations in the analytical model

Lagrangian equations of motion

Assumptions of two-dimensionality and incompressibility allow one to introduce a streamfunction:

\[ \nabla \times \mathbf{V} = \mathbf{0}, \quad \nabla \cdot \mathbf{V} = 0 \]

The Lagrangian equations of motion are then

\[ \frac{d\mathbf{X}}{dt} = \mathbf{V}(\mathbf{X}), \quad \frac{d\theta}{dt} = \omega(\mathbf{X}, t) \]

These equations have Hamiltonian form with the streamfunction playing the role of the Hamiltonian \( \phi(x, y, t) \equiv H(p, q) \). The streamfunction is assumed to consist of a steady background subject to a time-dependent perturbation

\[ \phi(x, y, t) = \phi_0(x, y) + \epsilon \phi_1(x, y, t) \]

1) quasiperiodic perturbation: \( \phi_1(x, y, t) = \phi_1(x, y, t/N) \)

2) Diophantine condition: \[ \phi_0(x, y, \omega/\omega'), i, j = 1, \ldots, N \text{ are sufficiently irrational} \]

3) Nondegeneracy condition (Russmann): \[ \langle \phi_0, \mathbf{V}, \omega \rangle = \tilde{\omega} \]

Surviving tori (barriers for transport)

\[ \Delta \omega \propto \left( |\epsilon| |\omega'| \right)^{1/2} \]

Waves are shown in green on the upper left subplot. Note that it serves as a transport barrier for the color-coded trajectories near the shearless trajectory is shown in green. All trajectory curves. The shearless torus is shown in green. All trajectories are regular (nonchaotic) curves. No chaotic trajectories are present.

KAM theorem

1) quasiperiodic perturbation: \( \phi_1(x, y, t) = \phi_1(x, y, t/N) \)

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Simulations in the analytical model

\[ \Delta \omega \propto \left( |\epsilon| |\omega'| \right)^{1/2} \]

low values of shear

\[ \Delta \omega \propto \left( |\epsilon| |\omega'| \right)^{1/2} \]

small resonance widths

resonances less likely to overlap

surviving tori (barriers for transport)


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Remarks

- Transport is qualitatively different for small and large perturbation to the background.
- For a small perturbation: 1) the transport barrier is broken; and 2) both homoclinic and heteroclinic intersections of manifolds are present. It is the presence of heteroclinic intersections of manifolds that makes gyre-to-gyre-to-gyre transport possible.

References