Using Kalman smoother to derive daily gravity field solutions from GRACE L1B data

Enrico Kurtenbach, Torsten Mayer-Gürr, and Annette Eicker

Institute of Geodesy and Geoinformation, Department of Astronomical, Physical and Mathematical Geodesy, University of Bonn, Germany

Abstract

Different GRACE data analysis centers provide temporal variations of the Earth’s gravity field as monthly, 10-daily or weekly mean fields. These solutions are derived independently for each time span, i.e., no correlation between the analysed batches is considered. Following this procedure, an increase in temporal resolution is accompanied by a loss in accuracy. To avoid this problem, KURTENBACH et al. (2009) presented a new approach, which takes into account the temporal correlations of the gravity field variations thus enabling the enhancement of the temporal resolution up to daily snapshots. The GRACE Level-1B (L1B) instrument data processing is performed within the framework of a Kalman filter estimation procedure. In this contribution an improved approach is presented, which takes into account the full temporal and spatial correlation pattern of the expected gravity field signal. The required information in terms of an empirical auto-covariance function is derived in this approach from atmospheric, oceanic, and hydrological model data.

The Earth’s temporal gravity field as linear dynamic system

Observation model

Analyzing GRACE observations leads to a system of observation equations

\[ A_0 x_t = y_t + v_t \]

with the design matrix \( A_0 \), the vector of unknowns \( x_t \), the observations \( y_t \) and the noise vector \( v_t \sim \mathcal{N}(0, R) \).

Least squares adjustment leads to a system of normal equations

\[ N_x = n_t \]

can be solved separately for each epoch \( t \).

Combining observation model and process model — the Kalman filter and smoother

The common tool to combine the information from observations \( y_t = A_0 x_t \) here from GRACE, and from a process model \( x_{t+1} = B x_t + w_t \) is the Kalman filter, which provides an optimal weighting of both in a least-squares sense. Because of the postprocessing analysis of the GRACE observations, not only a filter but also a fixed-interval smoother [RAUCH et al., 1965], is applied to use all observations in the given interval \( t \in [0,\ldots,7] \) when estimating the state at epoch \( t \).

M1 — Random walk

KURTENBACH et al. (2009) used the simple assumption for the process dynamic

\[ B = I \]

where the prediction error can be modeled as covariance matrix of the first differences of the considered dynamic process: \( Q = C \{(x_{t+1} - x_t)\} \).

Instead of the isotropic covariance function used by KURTENBACH et al. (2009), in this study a full covariance matrix \( Q \) of the prediction error is derived from geophysical models.

M2 — Least squares prediction

The full correlation pattern between two states of a stationary, first order Markov process can be described by

\[ C \{(x_{t+1} | x_t)\} = \Sigma \Sigma_x \Sigma \Sigma \]

According to MORRITZ (1980) a linear least-squares predictor \( x_{t+1} = B x_t \) can be found as

\[ B = \Sigma_x \Sigma^{-1} \]

with the covariance matrix of the prediction noise \( w \)

\[ Q = \Sigma - \Sigma_x \Sigma \Sigma_x \]

The Earth’s gravity field is modelled as a linear, discrete-time dynamic system.

Process dynamic

Given the dynamic \( B \) and process noise \( Q \) for M1 (random walk):

\[ \hat{x}_t = B \hat{x}_{t-1} + \Sigma \]

\[ \Sigma_{t+1} = \Sigma - \Sigma_x \Sigma \Sigma_x \]

for M2 (least squares prediction): \( \Sigma = C \{(x_{t+1} | x_t)\} \) and cross-covariance \( \Sigma_x = C \{(x_{t+1} | x_{t+1})\} \) from AOH of 01/1976 to 12/2006.

Run process \( x_{t+1} = B x_t + w \) with random prediction noise \( w \sim \mathcal{N}(0, Q) \) for \( T = 1000 \) days

Analysis of simulated GRACE observations

The Kalman smoother \( \hat{x}_t = \text{AOH}(t) \) for \( M2 \)

Simulate dynamic process

Using Kalman smoother to derive daily gravity field solutions from GRACE L1B data

The presented approach M2 was used to derive daily solutions of degree/order 40 in the current GRACE gravity field release ITG-Grace2010 of the IGG at University of Bonn. These daily solutions were also used as an improved dealiasing product when deriving the monthly solutions. For further information see http://www.igg.uni-bonn.de/apmg/index.php?id=itg-grace2010

Contact:

Enrico Kurtenbach
kurtenbach@igg.uni-bonn.de

Nussallee 17
D-53115 Bonn
www.igg.uni-bonn.de/apmg/