The Influence of Gravity on the Evolution of the Kelvin–Helmholtz Instability around Venus

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Abstract

The solar wind flow around the ionospheres of unmagnetized planets induces various processes at the boundary layer between the solar wind and ionospheric plasma. In principal, such a flow configuration is thought to be unstable with regard to the Kelvin–Helmholtz instability. Observations of Pioneer Venus Orbiter (PVO) gave rise to the idea that this instability might lead to the formation of so-called plasma clouds, which consist of ionospheric particles and thus could contribute to the planetary loss (Brace et al., 1982). Recently, the magnetometer of Venus Express (VEX) observed vortices in the magnetic field, giving again rise to the question of the origin of such structures (Pope et al., 2009).

We present a numerical study of the 2D Kelvin–Helmholtz instability and its vortices, where an initial plasma configuration appropriate for the situation around unmagnetized planets is assumed. We solve the set of ideal MHD equations numerically with the TVD Lax–Friedrichs algorithm. Our density profile is such that the mass density increases toward the lower plasma layer (i.e. the ionosphere). A dense ionosphere leads to smaller growth rates of the instability and thus has a stabilizing effect for the boundary layer. Moreover, we include source terms in the equations, enabling us to study the influence of gravity.

Our results show that gravity affects the evolution of the KH instability. However, the effect is not very significant. We thus conclude that the density increase towards the planet stabilizes the boundary layer around Venus more than gravity.

Equations and Initial Profiles

We solve the ideal MHD equations in 2 dimensions, using the so-called Lax–Friedrichs TVD scheme. We also include a source term to study the influence of gravity. This term is calculated using the 4 stage Runge–Kutta method.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) + \Pi = 0
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla P + \mathbf{f} + \mathbf{B} \times \mathbf{v} + \rho \mathbf{g}
\]

Initial profiles:

- B perpendicular \( \mathbf{v} \)
- \( v_i(y) = 0.5 \rho_0 (1 + \tanh(y)) \)
- \( \rho(y) = 0.5 (\rho_0 + \rho_1) + (\rho_0 - \rho_1) \tanh(y) \)
- \( B(y) = 0.5 B_0 (1 + \tanh(y)) \)

Initial pressure has to fulfill the condition

\[
\Pi(y) = -\rho(y)g \]

which leads to

\[
\Pi(y) = \Pi_0 - \rho_0 \rho_1 y + \frac{\rho_0 - \rho_1}{2} \ln(|\sinh(y)|)
\]

For the numerical procedure, we normalize the plasma parameters with the magnetosheath values (\( \rho_0, \rho_1 \)) and the spatial scales with the boundary half-width \( a \).

For this study, we take: \( \rho_0 = 1.0, \rho_1 = 1.0, B_0 = 1.5, \Pi_0 = 3.0 \). The ionospheric density \( \rho_1 \) is chosen to be \( \rho_1 = 10 \) and \( \rho_1 = 100 \), respectively. We compare solutions for different values of the gravity \( g \), varying between \( g = 0 \) and \( g = 12 \times 10^{-3} \).

Results

It is well known that at the beginning, the perturbation grows exponentially and a linear growth rate can be determined. We made calculations for different wave numbers \( k \) to find the maximum growth of the instability.

Fig. 1 shows the linear growth rates for \( \rho_1 = 10 \) and different gravities, as well as for \( \rho_1 = 100 \). We can see the stabilizing effect of gravity. There seems to be a slight shift of the maximum to larger wave numbers for increasing gravity. Interestingly, our simulations show that there is hardly any effect of gravity for the case \( \rho_1 = 100 \). We see is the strong stabilization due to the large density jump across the boundary layer and a shift of the maximum growth rate to smaller wave numbers.

Fig. 2 shows the evolution of the maximum velocity \( v_i \) for the same cases. We can see the linear phase at the beginning and the nonlinear phase after saturation.

Discussion and Conclusions

This work follows a recent study of the KH instability at unmagnetized planets by Amerstorfer et al. (2010), where they studied the stabilizing influence of a density increase towards the planet. From theory, it is well known that gravity also exhibits a stabilizing effect on the boundary layer. We wanted to address the question, how effective the stabilization due to gravity is compared to the density increase. We present numerical simulations with the same initial conditions and numerical algorithm as in Amerstorfer et al. (2010), but we include a source term, containing gravity, on the right hand side of the MHD equations.

Our results show that gravity affects the evolution of the KH instability. However, the effect is not very significant – especially if we bear in mind that we took values for gravity which are at least one order of magnitude larger than what should be taken for Venus.

Thus, we conclude that the most important stabilizing effect for the KH instability at boundary layers around unmagnetized planets is the density increase towards the planet.

The KH instability has been discussed as a loss process for planetary ions by various authors (e.g. Brace et al., 1982; Amerstorfer et al., 2010). However, the strong stabilizing effect of the density increase at the ionopause should prevent the continuous development of vortices – and thus the production of plasma clouds. If plasma clouds are the result of KH vortices, they are more likely to form in low density regimes with lower density jumps, as for example the induced magnetopause at Venus. This would mean that the plasma clouds would not contain a dense ionospheric plasma, and thus the loss rate would probably be lower than previously assumed.

Anyway, it is also possible that the detached plasma structures, observed above the Venus ionopause by PVO, are produced through other processes than the KH instability. We hope that the extended VEX mission will provide us with more observations which will shed light on this issue.

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References